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NOVICES VIEWS ON RANDOMNESS

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Novices and experts rated 18 phenomena as random or non-random and gave justifications for their decisions. Experts rated more of the situations as random than novices. Roughly 90% of the novice justifications were based on reasoning via a) equal likelihood, b) possibility, c) uncertainty, and d) causality.

Much of the prior research on randomness has focused on people's ability to generate and identify strings of random characters (Falk, 1981; Wagenaar, 1972). The major finding has been that people hold non-normative expectations about the production of random strings. For example, a random sequence of heads and tails typically contains longer runs than people expect would occur by chance. These studies have recently been criticized on a variety of accounts, including the argument that since a random sequence cannot be rigorously defined, it makes little sense to speak of people's inability to generate one (Ayton, Hunt, & Wright, 1989).

"Randomness," in fact, comprises a family of concepts. In this study we explore in particular the use of the word as it is used in the phrases, "random phenomenon," "randomizing device," and "random sample." In this sense, randomness is a collection of abstract models which can be applied to various situations. Sometimes we identify these models closely with some physical system, like a coin toss, or blind drawings from an urn filled with balls. In actuality, such physical systems are imperfect instantiations of some "ideal" random-generating system that is only realized in the abstract. Thus, we don't talk about flipping a coin, but flipping a "fair" coin.

Randomness, as an application of an ideal model to some phenomenon, is best thought of as an orientation we take toward, rather than as a quality that belongs to, the phenomenon. This meaning is inherent in the notion of a model. When we apply a model to some situation, we do not regard the model as isomorphic to the target situation as a whole, but only to certain aspects of the situation. This view of randomness explains why most experts are not bothered by the idea of "psuedo-random" numbers. These numbers are produced in perfectly determined ways, yet remain unpredictable to those who do not know the seed and multiplier used to produce a particular sequence. Such a system is not random, except in regards to the orientation adopted by the observer.

While admitting that the notion of randomness is ambiguous and complex, we maintain that variants of the concept are nevertheless at the heart of probabilistic and statistical thinking, and that people's beliefs about randomness must be figured into attempts to teach these topics (Falk, 1991; Falk & Konold,in press; Pollatsek & Konold, 1991). In this article we present preliminary results of an exploratory study of people's subjective criteria of randomness. We asked both novices and experts to categorize a variety of situations as either random or not random, and to give rationales for categorizing each situation. Our primary objective was to identify, in the justifications of the novices, defining features of random and non-random situations.

Some potentially critical features of randomness for the novice have been suggested by Nisbett, Krantz, Jepson, and Kunda (1983), who found that subjects are more likely to employ statistical reasoning to an event when it a) involves a repeatable process with a finite set of symmetric outcomes (e.g., rolling a die), b) consists of outcomes that are produced via a mechanism that is associated with chance (e.g., blindly drawing from a set of well-mixed objects), and c) has been identified within the culture as largely unpredictable and capricious (e.g., the weather).

Method

Twenty subjects (twelve women and eight men) were recruited from undergraduate psychology courses at the University of Massachusetts. Subjects were given 18 cards on each of which was written a brief description of some situation (see Table 1) and were asked to sort the cards, one at a time, into "random" and "non-random" piles. After placing a card in a pile, they were asked to give a brief justification for their categorization. The sessions were videotaped. The same sorting task was given to five experts, four of whom teach graduate-level statistics in psychology departments; the other is a statistician.

Results and Discussion

Randomness Judgments

A basic question is whether salient features of the items were predictive of subject categorizations. Table 1 shows the percentage of novices and experts that categorized each item as random. Experts categorized more items as random than the novices (62% compared to 53%). The largest differences occurred with situations involving real-world phenomena. For example, 80% of the experts judged Item 12, which involved occurrences of earthquakes, as random compared to 20% of the novices.

Item	Group	
	Novice	Expert
1. Whether or not a planted seed germinates.	35	40
2. The number showing up on a die that has already been rolled but that you can't see.	95	80
3. The number of tomatoes you get in your serving of tossed salad at a restaurant.	35	40
4. The winner(s) of next week's megabucks state lottery.	95	100
5. Selecting one of a variety of available flavors of ice cream given that the stranger in the line in front of you is doing the selecting.	80	40
6. Selecting one of a variety of available flavors of ice cream given that you are doing the selecting.	5	0
7. The number of heads that occur in 100 tosses of a fair coin.	85	100
8. Dividing a group of players into two basketball teams such that one team is not obviously better than the other.	0	20
9. The next gear a car with 5 speeds is shifted into given that it is currently in 4th gear.	20	50
10. Whether or not it rained in Amherst on April 3, 1936.	45	50
11. Whether it will rain tomorrow in Amherst.	35	60
12. Whether a large magnitude earthquake occurs in Boston before one occurs in Los Angeles.	20	80
13. Picking a white marble from a box that contains 10 black and 10 white marbles.	100	100
14. Picking a white marble from a box that contains 10 black and 20 white marbles.	70	100
15. Saying the first thing that comes to your mind.	30	40
16. Whether or not you get the flu in the next month.	40	80
17. Whether or not you get exposed to the flu in the next month.	65	40
18. The outcome of the fifth flip of a fair coin that has landed with heads up on the previous four flips.	100	100

Table 1. Percentage of novices and experts who rated each item as random.

The items can be grouped into "real" (Items 1,3,5,6,8-12,15-17), and "stochastic" situations. The stochastic items correspond roughly to those that Nisbett et al. (1983) would rate high on their three features as summarized above. Table 2 shows the mean percentage of items of each type that were rated as random, along with the standard deviations over subjects. As can be seen in Table 2, a higher percentage of stochastic items than real items were classified as random by both experts and novices. This is not surprising given that many of the real items were chosen because they seemed characteristically non-random.

Item type	Group			
	Novice		Expert	
	Mean%	SD	Mean%	SD
Real	37.3	22.4	43.3	33.0
Stochastic	90.8	14.8	93.3	9.1
Symmetric	97.5	11.2	95.0	11.2
Non symmetric	77.5	34.3	90.0	22.4

Table 2. Mean percentage of random ratings by experts and novices as a function of item type.

The stochastic items were further broken down into those with symmetric outcomes (2,4,13,18) and non-symmetric outcomes (7,14). This feature seemed to make little difference in the categorizations of the experts. However, the novices were more likely to rate a stochastic situation as random when its outcomes were symmetric (97.5%) than when they were non-symmetric (77.5%). This finding is born out in the analysis of subjects' justifications.

Analysis of Justifications

Subject justifications were transcribed from the videotapes, and various response categories were developed to capture basic rationales that were used repeatedly by novices. Table 3 shows the number (and percentage) of justifications of the various types for both the novices and experts. Below we describe these categories and provide examples from the transcripts.

Justification	Group	
	Novice	Expert
Equally-likely	64 (17.2)	3 (3.3)
Possibility	63 (16.9)	1 (1.1)
Uncertainty	82 (22.0)	25 (27.2)
Causality	128 (34.4)	20 (21.7)
Model	11 (2.9)	17 (18.5)
Other	24 (6.5)	26 (28.3)

Table 3. Number (and percentage) of various types of novice and expert justifications.

Equally likely. According to the "equally-likely" justification, a phenomenon is random only when each of its outcomes have the same probability. This reasoning, which mirrors early historical development (Zabell, 1988), is exemplified by responses of Subject 9 on Items 13 and 14. Brief item descriptors appear in parentheses.

13. (10/10) "Random. You have an equal chance of getting white or black."
14. (10/20) "Not random. You have a greater chance that you'll pick white."

This reasoning, used rarely by the experts, was used by novices to justify 17% of their categorizations, and was not limited to stochastic items. For example, Subject 6 categorized Item 1 (Seed) as random because, "Each seed has an equal chance of growing or not growing." Subject 13 categorized Item 9 (Gear) as not random because: "Usually you are going to go to a 5th or a 3rd. First and second don't have the same chance."

Multiple possibilities. According to the justification of "multiple possibilities," a phenomenon is random when there is more than one possible outcome and is not random when there is only one possible outcome. In justifying a "random" categorization, subjects typically noted that any of the multiple outcomes were possible. Responses by Subject 6 are shown below as examples.

9. (Gear) "Not random. Has no choice – it has to go into 5th gear."
11. (Rain tom.) "Random. It may or it may not."

Justifications based on possibility were rare in the case of the experts (only one instance). This reasoning, as well as the equally-likely rationale, may be related to an informal interpretation of probability that has been described as the "outcome approach" in prior research by Konold (1989a; 1989b).

Uncertainty. According to the "uncertainty" justification, a phenomenon is random when there is no prior knowledge about the outcome, and thus no ability to predict. When prediction is possible, the phenomenon is non-random. This justification, exemplified below by responses of Subject 20, was used in 22% of the novice and 27.2% of the expert categorizations.

10. (Rain '36) "Non random, because there is a way to predict the weather."
18. (5th flip) "Random. There is just no way to determine what is going to happen."

Causality. According to this justification, situations are random when no causal factors can be identified, and thus there is no potential to control the result. If causal factors are present, and/or control is possible, the situation is considered non random. For the novices, this was the most commonly-used justification (34.4%), and was also used frequently by the experts (21.7%). The examples below are statements made by Subject 18:

1. (Seed) "Not random, because it depends on soil and all kinds of other things."
7. (# Heads) "Random, because I have no control over what the coin is going to do."

The four categories of justification described above were developed on the basis of analyses of the novice justifications, and for this reason account for a higher total percentage of the novice than the expert justifications (90.6% vs 53.3%). Based on a separate analysis of the expert justification, we added a fifth rationale, as described below.

Model. By this reasoning, the randomness of a situation is established by comparing it to some standard model of randomness. In the case of Expert 3, situations were frequently compared to a "box model."

4. (Lottery) "Random. It is determined by a random device, or a pretty good approximation of one."
5. (Stranger ice) "Non random. He does it by some kind of rule, unknown to you, but you don't have any serious box model."

As might be expected, the experts used this rationale more frequently than the novices (18.5% compared to 2.9%). However, even with the addition of this response category, roughly 28% of the expert justifications did not fit into any of the five categories. Several of the experts expressed their dissatisfaction with having to categorize items as either random or not random. They tended to view randomness as an entity that can be present in degrees, rather than as a categorical attribute, and described several of the situations as consisting of both random and non-random components.

Indeed, an important idea in statistics is the notion that scores or measures can be decomposed into two sources of variation: systematic (explained), and random (unexplained). One of our objectives in future analyses of these data is to identify aspects of novice thinking that present barriers to the development of this "component" view of phenomenon. Subject reliance on "possibility" and "equal-likelihood" are two possible barriers that we are currently exploring. Seeing randomness in terms of possibility might lead students to overgeneralize the concept, viewing any situation as random as long as there is more than one possible outcome. On the other hand, reliance on equal-likelihood restricts the notion of randomness. Introducing students to a wider range of probabilistic situations, including ones in which outcomes are not equally likely, is an approach we are currently testing which may help students develop probabilistic intuitions that can be successfully transferred to statistical thinking.

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