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ABSTRACT

A number of studies have reported that there is a strong tendency to ignore base-rate information in favor of individuating information, except when the former can readily be incorporated into a causal schema. In this study, students in eight undergraduate classes were given problems in which the base-rate information was either causal or noncausal and either incongruent or congruent with the individuating information. In addition, 12 subjects were interviewed as they attempted to solve several versions of one of the problems. The results indicated: (1) strong individual differences in the perceived importance of base-rate information and even in how the probability estimation task itself was interpreted; and (2) greater use of base-rate information congruent with the individuating information than of base-rate information which is incongruent. The interview data indicate that it is difficult to determine from the answer alone whether or not the subject thought that the base-rate information was relevant. These data also suggest that subjects have different strategies for dealing with probability estimation problems. (Author/TW)

PROBABILITY ESTIMATION AND THE USE AND NEGLECT OF BASE-RATE INFORMATION

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Abstract

A number of studies have reported that there is a strong tendency to ignore base-rate information in favor of individuating information, except when the former can readily be incorporated into a causal schema. In the present study, students in eight undergraduate classes were given problems in which the base-rate information was (1) either causal or noncausal and (2) either incongruent or congruent with the individuating information. In addition, twelve subjects were interviewed as they attempted to solve several versions of the one of the problems. We found (1) strong individual differences in the perceived importance of base-rate information and even in how the probability estimation task itself was interpreted, (2) little if any effect of the causality manipulations employed by Ajzen (1977) and Tversky and Kahneman (1980), and (3) greater use of base-rate information congruent with the individuating information than of base-rate information which is incongruent. The interview data indicate that it is difficult to determine from the answer alone whether or not the subject thought that the base-rate information was relevant. These data also suggest that subjects have different strategies for dealing with probability estimation problems. One of these we characterize as not only nonBayesian, but also nonprobabilistic.

Probability Estimation and the Use and Neglect of Base-Rate Information

Consider the following problem:

> A cab was involved in a hit-and-run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following data:
>
> (i) 85% of the cabs in the city are Green and 15% are Blue.
>
> (ii) A witness identified the cab as a Blue cab. The court tested his ability to identify cabs under the appropriate visibility conditions. When presented with a sample of cabs (half of which were Blue and half of which were Green) the witness made correct identifications in 80% of the cases and erred in 20% of the cases.
>
> Question: What is the probability that the cab involved in the accident was Blue rather than Green?

The above problem, and others similar to it, have been administered to hundreds of subjects (e.g., Bar-Hillel, 1980; Kahneman and Tversky, 1972, 1973; Lyon and Slovic, 1976; Tversky and Kahneman, 1980) in order to understand how people arrive at probability estimates when given both evidence specific to the case (here the witness information) and base-rate information (here the color distribution of cabs in the city). Below are four brief excerpts from interviews we conducted with undergraduate students who were asked to consider the problem.

> I guess it could very well be a blue cab. But I would probably guess that it would be a green one, because there were 85% green. There's more of a probability of it being green than it is of being blue.
>
> There's still a 15% chance that the other cabs were causing them, so it doesn't mean that they couldn't cause one. It's 50-50.

> The question is kind of open-ended, and you have these two things that you can't really put together--85% of cabs are green and 15% are blue. So according to that, the probability could be 15% that it was involved. And then with the other thing it could be 80%. So I'm not really sure how to use this information to get a probability.

> It mattered less that there were 85% cabs...85% were green and 15% were blue. I'd assume that a person could identify the difference in a blue and green cab. And since he got it right 80% of the time...I would assume that this man...was right that it was a blue cab. But he did get it wrong 20% of the time, so I wouldn't think there'd be a 100% chance. So I guess, about three-quarters, (75%).

What can be concluded from such statements? Each seems very different and some sound very confusing to those well-versed in probability theory. In this paper, we will attempt to provide some insights into the ways subjects approach this and similar problems. But first we must provide a brief account of why questions like these have interested psychologists and how they have proceeded.

There has been a great deal of interest in how individuals make judgments. In order to understand how judgments are made, it must be determined which of the available sources of information are relevant and how the different pieces of relevant information are weighted and combined. The idea that strength of belief can be indexed by subjective probabilities and that Bayes' theorem provides a normative algorithm for revising these subjective probabilities given additional evidence, is central to Bayesian statistics (e.g., Raiffa, 1968; Savage, 1954) and has important potential applications for theories of social judgment (e.g., Nisbett and Ross, 1980). Therefore, a good deal of work has focussed on whether judgments in uncertain situations can be predicted by Bayes' theorem. The seminal work in the field (e.g., Edwards, Lindman and Savage, 1963) suggested that people might be conservative Bayesians. In those studies, subjects were presented with samples of data drawn from one of several possible sources and were asked to estimate the probability that a particular source had generated the data. Probability estimates were found to take account of the sample data observed but to remain closer to the prior probability of the source than would be predicted by Bayes' theorem. Lyon and Slovic (1976) have pointed out, however, that later analyses showed the conservatism in these tasks did not result from overweighting the prior probabilities, but rather from improper operations performed on the sample data.

The normative solution to the cab problem mentioned above employs Bayes' theorem. If we use lower-case letters to indicate witness reports and upper-case letters to indicate actual color of cab, what is asked for in this problem is an estimate of P(B/b), the probability that given the witness report of blue, the cab eliciting the report was actually blue. In odds form, Bayes' rule can be written

Posterior odds in favor of a particular inference = Likelihood ratio for that inference x Prior odds in favor of that inference

$$\frac{P(B/b)}{P(G/b)} = \frac{P(b/B)}{P(b/G)} \times \frac{P(B)}{P(G)}$$

$$= \frac{.8}{.2} \times \frac{.15}{.85} = \frac{12}{17}$$

so that $P(B/b) = \frac{12}{12+17} = .41$

Another correct approach to this problem that appeals to different intuitions is to first calculate the appropriate joint probabilities as indicated in Table 1. It follows from Table 1 that

$$P(B/b) = \frac{P(B \cap b)}{P(b)} = \frac{.12}{.12+.17} = .41$$

Insert Table 1 about here

A number of studies have reported that for Bayesian inference problems like the cab problem, subjects do not act like Bayesians, but rather appear to ignore the base-rate information in making their judgments (e.g., Bar-Hillel, 1980; Casscells, Schoenberoer, and Graboys, 1978; Hammerton, 1973; Kahneman and Tversky, 1973; Lyon and Slovic, 1976). It has also, however, been reported that base-rate information does influence probability judgments if it can be interpreted as being causally related to the target outcome (Ajzen, 1977; Tversky and Kahneman, 1980). The base-rate information in the cab problem would be considered to be causal if, for example, the problem stated that there were equal numbers of blue and green cabs but that the latter were involved in more accidents. This information would be expected to elicit the causal explanation that the drivers of green cabs are involved in more accidents because they are more reckless or less competent. Another example, taken from Ajzen (1977), involves the prediction of success on an exam for a particular student for whom a description is given. Information about the percentage of students passing the exam is considered to be causal because it permits the respondent to infer the difficulty of the exam, which presumably has a causal effect on success or failure.

Our interest in problems of this type arose while teaching conditional probability and Bayesian logic to psychology graduate students. We soon became aware that students were not intuitive Bayesians When the following example was given in class,

> A medical team set up several public clinics in a large city to help in the early detection of cancer. The cancer test they used was 95% reliable, meaning that 95% of the people who had cancer got a positive result on the test and 95% of the people who didn't have cancer got a negative result. In that city it is known that 1% of the people have cancer and everybody in the city was eventually tested.
>
> What are the chances that a person who receives a positive test actually has cancer? (Express your answer as a percentage.)

we found that a large majority of students gave 95%, the hit rate of the test, as their answer (the correct answer is 16%), despite the fact that the base-rate information here is causal. Moreover, many of the students found it difficult to understand why the correct answer was not 95% even after seeing the correct solution worked out.

We wanted to know how individual students viewed the relevance of base-rate information, and how, if they considered it to be relevant, they thought this information should be combined with the other information in the problem. From our point of view, the usefulness of previous research on Bayesian problems was limited by the fact that it has depended almost exclusively on questionnaire data and has, with few exceptions (notably, Bar-Hillel, 1980), reported only measures of group performance such as modal or median response. Inferring from such measures to individual thought processes is hazardous, especially since we know from our own work and from the few papers in which distributions of responses have been made available that the modal or median response usually accounts for less than half the respondents. It is also difficult to infer a strategy from a numerical answer. We have observed subjects with markedly different strategies arrive at the same numerical answer and subjects with identical strategies arrive at different answers. With the fairly complex Bayesian problems discussed here, it seems entirely possible that wrong answers could have a number of causes, including such uninteresting ones as the subject misreading or being overwhelmed by the problem. Consequently, in the current study we not only administered questionnaires in which several problems were systematically varied but also conducted a number of in-depth interviews in which subjects were encouraged to talk aloud as they reasoned through several versions of a problem.

More substantively, our classroom experience with the cancer problem (in which the base-rate information was causal yet frequently ignored) led us to be somewhat skeptical of the salience of causality for this kind of problem. Accordingly, we wished to collect data on several versions of this problem. Secondly, we wanted to study problems such as the cab problem for which causal and noncausal versions had been constructed (Tversky and Kahneman, 1980), so that we could determine whether subjects viewed causal base-rate information in a fundamentally different way than noncausal base rates. Thirdly, while we had noticed that subjects almost invariably gave the answer 95% to the cancer problem described above, when asked what the chances of cancer were given a negative test, the answers were more varied. We thought that some subjects might view the problem in a fundamentally different way when the individuating information and the base rates were congruent (i.e., both pointing to the same conclusion, in this case, the person not having cancer) than when the individuating information and the base rate were incongruent, even though we knew Lyon and Slovic (1976) had not found any effect of direction of base rates on the median response to the cab problem. Acco dingly, we employed problems in which the base rates were either congruent or incongruent with the individuating information.

Method: Questionnaires

Variants of three different Bayesian inference problems were administered in written form to students in eight undergraduate classes at the University of Massachusetts.

Cancer A. The cancer problem given in the introduction was included in the study with subjects asked to make responses to three questions:

> (a) Suppose you choose a person at random. Let's call him John. Without knowing any more about John, what do you think are the chances that he has cancer? (Express your answer as a percentage.)
>
> (b) Suppose that you are given the additional information that John just took the detection test and got a positive result. Now what do you think the chances are that he has cancer? (Express your answer as a percentage.)
>
> (c) Say a second person is chosen randomly (Fred) and takes the test. The test turns out negative. What do you think the chances are that Fred has cancer? (Express your answer as a percentage.)

Cancer B. Because we expected most subjects to respond to the (b) and (c) parts of the Cancer A problem with the hit rate and false alarm rate of the test, we developed a second version that was changed in two ways that we thought might result in smaller answers: (1) the base rate was reduced from 1% to .1% and (2) the questions were asked in a nonprobabilistic fashion. The revised problem was as follows:

> A medical team set up several public clinics in a city to help in the early detection of cancer. They gave the test to 100,000 residents. The cancer test they used was 95% reliable: 95% of the people who had cancer got a positive result on the test and 95% of the people who didn't have cancer got a negative result. It was later determined that of the 100,000 people, 100 people (.1%) actually had cancer.
>
> What percent of the people who got positive test results in fact had cancer? __________%
>
> What percent of the people who got negative test results in fact had cance . __________%

Taxicab A. This was the noncausal version of the cab problem that is given in the introduction (taken from Tversky and Kahneman, 1980). A subject responded by placing a mark along a scale that went from 0% - absolutely certain that it was Green to 100% - absolutely certain that it was Blue.

Taxicab B. This was the causal version of the cab problem taken from Tversky and Kahneman (1980). In this version the second sentence of the Taxicab A problem was replaced by

> Although the two companies are roughly equal in size, 85% of the accidents in the city involve Green cabs and 15% involve Blue cabs.

Taxicab C. This was a version of the cab problem that was both causal and congruent. In addition to reversing the direction of the base rates, the accuracy of the witness was changed from 80% to 70% in order to allow room for possible responses between the base-rate and witness accuracy figures

Exam A. This was the noncausal version of a problem used by Ajzen (1977). It reads as follows:

> Two years ago a final exam was given in a couse at Yale University. An educational psychologist interested in scholastic achievement interviewed a large number of students who had taken the course. Since he was primarily concerned with reactions to success, he selected mostly students who had passed the exam. Specifically, about 75% of the students in his sample had passed the exam.
>
> Gary W. was among the students interviewed. Gary was a person of average intelligence who had sometimes had problems in mastering class material. He had found the course quite boring and he had expended little time and effort in preparation for the final exam.
>
> Indicate on the scale below your judgment of the probability that Gary W. was among the students who passed, rather than failed the final exam.

Subjects indicated their judgment by placing a mark on a scale that went from 0% - absolutely certain that he failed to 100% - absolutely certain that he passed.

Exam B. This was the causal version of the exam problem and was also taken from Ajzen (1977). In this version, the first paragraph and the first sentence of the second paragraph of the Exam A problem was replaced by

> Two years ago, a final exam was given in a course at Yale University. About 75% of the students passed the exam.
>
> Gary W. was a student in the class.

Procedure

Each problem was typed on a separate page. Different combinations of problems were administered in the eight undergraduate classes as indicated in Table 2. All were introductory statistics classes with the exception of classes 5 and 7 which were Introductory Psychology and Cognitive Psychology, respectively. Students in statistics classes 1, 2, and 3 received the questionnaires during the first week of the semester, well before there had been any discussion of probability. Students in classes 4, 6, and 8 were given the questionnaires somewhat later. With the exception of class 3 in which a cancer problem was given on one occasion and a taxicab problem and an exam problem were given on a second occasion, no subject received more than two problems. No subject received more than one version of any problem. Each subject in classes 1, 2, 3, 4, and 5 received either the Taxicab A (noncausal) and the Exam B (causal) problem or the Taxicab B (causal) and the Exam A (noncausal) problem.

Subjects were verbally instructed that the problems contained some information that they were to consider in estimating the probability of a certain event. There was no time limit.

Insert Table 2 about here

Results and Discussion: Questionnaire Data

It is clear from the questionnaire data that few if any of our subjects were intuitive Bayesians. However, the pattern of responses to the different problems is complex and unlikely to be explained by any simple model. In particular, our data do not support the simple picture that causal base-rate information is incorporated into probability judgments while noncausal base-rate information is not. There is also reason to believe that some college undergraduates have difficulty with the instructions to estimate the probability or the "chances" of an event. For example, 107 subjects in their first week in introductory statistics courses were given the Cancer A problem in which base-rate information was stated explicitly, namely, 1% of the people in the city had cancer. The first part of the problem simply asked what the chances were (expressed as a percentage) that a randomly chosen person in the city had cancer. Only 74 of the 107 subjects gave the answer 1%. Of the remaining 33 subjects, three responded with .01, an answer that would be expected if subjects ignored the instructions to respond with a percentage and gave a proportion instead. The modal incorrect responses (seven respondents each) were 5% and 50%. The latter response is particularly interesting since we believe some subjects use it to indicate an extreme degree of uncertainty or lack of knowledge. We will discuss this issue further in the next section. In the remainder of this section, we will not consider the data of the subjects who gave an answer other than 1%.

Three general themes that emerge from the questionnaire data are (1) the pattern of responding differed across the different problems, with the lowest apparent use of base-rate information in the cancer problems despite the fact that these base rates were causal; (2) where causality was explicitly manipulated, the pattern of responses was about the same in the causal and noncausal versions of problems; and (3) the pattern of responding appeared to be different for versions of problems in which individuating information and base-rates were congruent than when they were incongruent.

First let us consider the data from the cancer problems. For the positive test result, the great majority of subjects given the Cancer A problem did seem to ignore base-rate information, only 11 of the 74 subjects giving a response smaller than 95%, the hit rate of the cancer test. Also, as expected, answers were more varied for the negative test result, with 44.6% of the answers at or below the base rate, even though the modal answer was 5%, the false alarm rate of the cancer test.

The modal answer for the Cancer B problem was also 95% for the positive test result. However, as can be seen from Table 3, there was a pronounced tendency for answers to be smaller. The mean response of 47.5% was significantly smaller ($t(109) = 4.76$, $p < .001$) than the mean answer of 83.4% given for the Cancer A problem and the distributions of answers for the positive test result were significantly different for the two problems ($\chi^2(2) = 19.13$, $p < .001$). These differences should be interpreted with caution, however, since the two problems were administered to different classes. The difference, however, at least suggests that the form of the problem may be important for the positive test result.

Insert Table 3 about here

There was virtually no evidence of intuitive Bayesian reasoning even for the 31 subjects who gave answers smaller than 95% in the positive test conditions. A subject might reasonably be considered to have demonstrated such reasoning in the cancer problems if he or she gave both a response between the hit rate of the test and the base rate for the positive test result and a response lower than the base rate for the negative test result. Only one of the 111 subjects whose data are represented in Table 3 met this criterion. A number of subjects apparently wished to use both base-rate and individuating information since they multiplied the two together or averaged them. There may have been others who wished to use both sources of information, but having no idea how to combine them, picked the single source of information that seemed most relevant.

It seems clear to us from looking through such data that simply obtaining numerical answers using a questionnaire has severe limitations as a tool for understanding how subjects deal with the complex problems used in this kind of research. For example, consider the subjects who apparently multiplied probabilities (five students gave answers of .95% or .095% and there were calculations made on the questionnaires in some cases). Why did they multiply them together? This might have shown some understanding, since one way of approaching the Bayesian calculation is by computing joint probabilities and then converting them to conditional probabilities (cf. Table 1). On the other hand, these subjects may have multiplied the probabilities merely because they knew that probabilities are sometimes multiplied, felt that some sort of calculation was expected, and did not know what else to do.

For the Taxicab and Examination problems, a major question was whether we could obtain findings similar to those of Ajzen (1977) and Tversky and

Kahneman (1980), showing the importance of whether base-rate information was causal or not. As can be seen from Tables 4 and 5, there was very little difference between the responses to the causal and noncausal versions of the two problems. For both problems, the mean posterior probability and the distribution of posterior probabilities were virtually identical in the causal and noncausal versions. While the average answer was a bit closer to the base rate for the causal version of the Taxicab problem (i.e., showing a slight tendency towards greater use of base rates), it was actually further from the base rate for the causal version of the Exam problem.

Insert Tables 4, 5, and 6

While there was little difference between causal and noncausal versions of the Taxicab problem, the pattern of responses was quite different from that of the positive test part of the Cancer A problem. In the latter, less than 15% of the responses were between the base rate and the individuating information, while for the Taxicab problem almost 50% of the responses were in this category. It is thus interesting to note that in this study causal base rates in two different problems produced strikingly different patterns of responses, while the causal and noncausal versions of the same problem produced virtually the same pattern of responses.

The fact that more than half of the answers given in the incongruent versions of the Taxicab problem (Taxicab A and B) were smaller than the witness accuracy of 80% does not necessarily mean that ,ase-rate information was used with much understanding. In the congruent version of the problem (Taxicab C), base-rate information was not only causal (85% of the accidents were caused by blue cabs) ' ıt pointed to the same color of cab identified by the witness. The witness was stated to be able to identify blue cabs as blue and green cabs as green with 70% accuracy. The appropriate intuition here would be that the answer should be greater than the maximum of 70% and 85%, since two pieces of information in the same direction should be better than either alone (the Bayesian answer is 93%). However, as can be seen from Table 6, of the 47 subjects given this version of the problem, only three gave answers greater than 85%. In fact, slightly fewer subjects gave responses larger than 70% than gave responses smaller than 70%, suggesting that some subjects may have been regressing to an implicit baseline of 50% rather than to an actual base rate of 85%.

Thus, to summarize, our data certainly confirm the hypothesis that few subjects are intuitive Bayesians, but are at variance with a picture now current (e.g., Nisbett and Ross, 1980) that subjects will almost uniformly ignore base rates except when they are causal. Most subjects did ignore base-rate information in certain versions of problems (e.g., the Cancer A positive test problem) but this was not the case for other problems. Moreover, we failed to replicate the finding that causal base rates are more readily incorporated into probability judgments. Although we cannot yet explain why our findings disagree with those reported by Ajzen (1977) and Tversky and Kahneman (1980), our data certainly indicate a lack of robustness in the phenomenon.

At this point in the research we could have tried other problems and versions of problems to explore our failure to replicate these studies. However, we felt that since inferences about subjects' thought processes from the pattern of answers on questionnaires were quite indirect, it would be desirable to conduct some in-depth interviews with subjects to understand better how they approach the complex kind of problem discussed here.

Method: Interviews

Twelve subjects were interviewed in order to explore in more depth the reasoning strategies employed in arriving at numerical answers for the cab problems. Subjects were given the Taxicab A (noncausal) and Taxicab B (causal) versions and then a congruent version of the problem which differed from Taxicab C only in that the witness accuracy remained at 80%.

Ten subjects were students enrolled in undergraduate psychology courses at the University of Massachusetts who were given extra course credit for their participation. Two subjects (#6 and #11) were former undergraduate students. Subjects were interviewed individually. The subject sat across the table from and faced the interviewer and was told that he or she would be given several problems in written form. Subjects were told to "think aloud" as they worked through the problems. A sheet of blank paper and a felt pen were available if they wished to do any calculations. All interviews were videotaped with the subjects' knowledge. The third author conducted all of the interviews. His experience with interviewing had begun about a year before these data were collected.

The interview proceeded in four general phases. In the first phase, subjects were given the problem and asked to read it aloud. Any misreading of the problem was corrected immediately. If the subject looked confused after the first reading, the interviewer suggested that he or she read through the problem again silently. If no response was given within approximately 30 seconds after reading the problem, the interviewer asked, "what are you thinking about?" The general strategy in this first phase was to allow subjects to provide an answer to the problem and to offer a rationale for the answer with as little probing from the interviewer as possible.

Once an answer was given and any spontaneous comments were made, the second phase began during which the interviewer encouraged subjects to talk as much as they could about the problem, the meaning of their answer, and the nature of the solution strategy. The majority of the interview time involved this type of questioning. Subjects were asked, for example, which pieces of information were used in arriving at their answers, why certain kinds of information were not used, and to explain how they arrived at the specific value they did.

The third phase involved changing parts of the problem in an attempt to gain further insights about the ways subjects thought about the problem. For example, subjects who had ignored the witness information might be asked what their answer would be if the witness had been 95% or 100% correct when tested.

In the fourth phase, the interviewer offered subtle challenges to their reasoning. For example, someone who ignored the base-rate might be asked, "what would you say to a person who argued that since there are more green cabs to begin with, they would still be more likely to have been involved in the accident?"

The interview, however, did not in all cases proceed easily from phase to phase. There was some mixture of probe types, especially within the last two phases. Also, subjects who felt very unsure of their answers were generally not given phase four probes. The interviewer attempted not to offer direct challenges to any subject's answer or rationale, or to suggest that the subject might have been in error.

The cab problems were the last problems in approximately an hour-long interview that included other questions about random sampling and probability. All three versions of the cab problem were given before proceeding into phases three and four of the interview. The Taxicab A (noncausal) version was given first. When the subject was given the Taxicab B (causal) version to read, he or she was first asked to comment on how this version differed from the previous (noncausal) version before attempting a solution.

After all problems had been given, subjects were debriefed and shown the correct solution to any problems that they desired.

Videotapes of the interviews were analyzed by all three authors. Descriptive and interpretive analyses were conducted as outlined by Konold and Well (Note 2).

Results and Discussion: Interviews

The answers given by the 12 subjects are given in Table 7. We attempted to use the interview data to understand more fully why subjects gave the answers that they did. In the following section we will try to indicate what we learned from the interviews, illustrating our points with excerpts from some of the protocols. Obviously, these excerpts must be kept brief and we are unable to convey fully the context for the comments made by subjects. We will make detailed protocols available to readers who are interested.

Insert Table 7 about here

Even though we believe that the interview data have given us considerable insight into how subjects approach the cab problems, we do not claim that subjects who were interviewed and subjects who received the problems on questionnaires approached them in exactly the same ways. For one thing, subjects who were interviewed spent more time on each problem and received several versions of the cab problem. Also, although subjects took readily to "thinking aloud" as they worked through the problems, the interview situation may have caused subjects to be somewhat more conservative. One difference between the questionnaire data (Table 4) and the interview data (Table 7) on the incongruent versions of the cab problem was that relatively more (almost half) of the subjects who received the problems on a questionnaire gave answers between 15% and 80%, suggesting the use of both base-rate and witness information. Several subjects who were interviewed wound up giving answers based solely on the witness information, even though they clearly wanted to combine the base-rate and witness information. They may have decided not to combine the two kinds of information because they felt they would have had to justify their combining procedure to the interviewer had they done so.

Effects of Base-Rate Causality.

We assumed that if the causality of base-rate information was important, the difference between the causal and the noncausal versions of the Taxicab problem should be salient. Eight of the 12 subjects interviewed did notice the change that had been made in the causal version without having to refer back to the page containing the noncausal version. Two subjects (#3, #8) initially thought that other changes had been made, and two (#3, #12) had to be told what the difference was, not having noticed it even after re-reading both versions.

Three subjects thought that the difference between the noncausal and causal versions was relevant to their answers. Only one subject (#11) actually gave a different answer for the causal version (50%) than for the noncausal version (80%), but she was not confident that there should be a difference.

> I: Can you maybe explain how these two pieces of information are different in your mind--why one is more relevant than the other?

> S11: Um...that's the problem. I don't really know...if... one is more relevant than the other. But...something about the fact that, they're talking about cab accidents here, that more accidents happen with the green cabs [14 sec. pause]..for some reason it just makes me feel like it would be more probable, that it was a green cab, even though 15% still involve blue cabs. [1]

Both subjects (#4, #6) who had initially responded to the noncausal version of the problem with the base rate gave the same answer to the causal version, but said their confidence in this answer was now different. Subject #4 said she was now less confident in her answer and indicated, "What's important is the number of cabs, not how many were in accidents." (We never succeeded in understanding her reasoning.) Subject #6 said her confidence was increased.

We have thus not found evidence, either from our questionnaire or interview data that causality of base-rate information is a potent factor in determining whether or not it will be used in making probability estimates. A failure to find an effect of causality of base-rate information with similar problems has also recently been noted by Karshmer (Note 1). Causality may be important in some populations and with some problems, but we do not see it as providing the basis for a general explanation of the use and neglect of base rates.

Rationale for Ignoring the Individuating Information.

Of the 12 subjects interviewed, only two failed to use the witness information in arriving at an answer. They did not seem to feel that they had adequate reason to trust the witness. Subject #4 said,

> I would probably guess that it would be a green one because there were 85% green. There's more of a probability of it being green that it is being blue...I mean a witness with 80% and 20% isn't very good. You can't say he was right, you can't say he was wrong. [2]

Subject #6, on being asked why the witness information had been neglected, answered,

> Well, it doesn't affect the probability...it doesn't have anything to do with the probability of which cab hit it--which color cab. It has to do with the probability of getting caught. Maybe you should take it into account, but it doesn't affect the probability... [3]

> ...if the question is asking for statistical probability, then it's...15% and this other information doesn't affect it. [4]

Later, she again defended her choice of green being more probable,

> ...because witnesses are notoriously wrong, and I'd rather go with the statistics. [5]

Both subjects seemed to view the fact that the witness made some errors as grounds for ruling out the witness information entirely.

Subject #4 was even unsure whether she would use the witness information if the witness had been 100% correct under the test conditions,

> I: ...you'd still have some doubts even if it was 100%?
>
> S: Yeah, I probably would...I don't know. Good thing I'm not a judge. [6]

Rationale for Not Using Base-Rate Information

Eight of the 12 subjects gave answers which, after probing, were judged to have been arrived at independently of the base-rate information. Subjects regarded the witness information as more relevant and viewed the witness as quite accurate. Some comments conveying this are as follows:

> S#3: It mattered less that there are 85% cabs--85% were green and 15% were blue. I'd assume that a person could identify the difference in a blue and green cab. And since he got it right 80% of the time...I would assume that this man...was right that it was a blue cab. But he did get it wrong 20% of the time, so I wouldn't think there'd be a 100% chance. So, I guess, about three quarters. [7]

> S#7: It seems to me, if he was only guessing he would only be 50% right. So, it seems that he, to me, that he's pretty good at identifying what cab it was. So, um, I don't know if it would be as high as 80%, because he still could be guessing a little bit. [8]

S#9: But to see a big, big cab sticking in front of you, and uh, you're not going to be blind enough not to see that it's blue. [9]

and later,

S: ...Well there's more green than there are blue, so you could say, well, since there are more green, there's a better chance for the green to get in an accident than the blue. But...I would say the blue, because of the man who'd seen the blue. His percentage of, uh, identifying the blue was, you know, 80%, which is very high out of 100%. And I would go by that... [10]

or,

I: This information (base rate) then, is not relevant to the problem?

S#10: No. What's relevant is what color he saw, not how many there are. [12]

and later,

S: ...but that doesn't have anything to do with it. That would be biasing your opinion based on what the cab, what cabs were in accidents, not based on true fact. The true fact is his his ability to see the color, which is 80%. [13]

Several subjects in this category spontaneously mentioned that had they been given no witness information, they would have used the base-rate information to estimate the probability.

S#1: At first, if I just read up to here (points to statement (i) in problem) I would say it was a green cab. [14]

I: Why?

S: Because...such a big quantity of cabs are green and so if there was an accident, I would just think it was a green one, because they are the most apt to get in the accidents. But when she was right 80% of the time,... and 20% of the time she was wrong, I'd say she was right. Because...she was right most of the time. [15]

S#7: If I just had to say off the top of my head,...if it wasn't a witness involved, I mean, I would probably assume it would be a green cab. But since a witness did--has an 80% chance of identifying the right color, which I assume would mean, basically, he could identify a blue cab, that it was still blue. I think 80%. [16]

On the surface one might conclude that those ignoring the base-rate information were reasoning in a very different manner than those ignoring the witness information. The interview data suggest that the reasoning employed may have been very similar: one of the information sources was assessed as being more relevant or reliable than the other, and the answer was based entirely on this source of information.

Several subjects responding with the answer 80% were reluctant not to include the base-rate information in their answer, but left it out entirely since they did not know how to combine it with the witness information.

Subject #11 was very reluctant to give any answer because,

> S: The question is kind of open ended, and you have these two things that you can't really put together--85% of cabs are green and 15% are blue. So, according to that, the probability could be 15% that it was involved. And then with the other thing it could be 80%...So I'm not really sure how to use the information to get a probability. [17]

She finally decided to go with the witness information since she was "not really sure if the first one (base-rate) is relevant--I'm pretty sure the second one is relevant."

Subject #12 gave 80% as an answer, but she qualified her answer by saying that there might be a way to combine the information, but that she didn't know how to do it.

> S: ...I seem not to be able to think of those two pieces of information as fitting neatly together into a way to solve it... [18]

Finally, we had wondered in looking at the questionnaire data how some subjects arrived at answers greater than 80% for the incongruent versions of the cab problem. We had suspected that such responses were the result of misreading the problem or were just careless responses. Subject #2, however, demonstrated how an answer based entirely on the witness information could be greater than the witness accuracy. He reasoned that, "...the witness is, in his mind, 100% sure that it was a blue cab. But the test that they gave--the probability of it being a blue cab might be brought down some..." So he averaged the 100% with the 80% test accuracy to get an answer of 90%.

Rationales for Intermediate Answers

Two subjects gave answers that were not based solely on the witness or base-rate information. Subject #8 gave 50% as an answer. His rationale was very straightforward, "I don't see any 100% scores on this so it's 50-50. There's a probability for error--50-50." He seemed to have a three-step function of probability. Either something never happened, in which case the probability was 0, something always happened, in which case the probability was 100%, or something sometimes happened--"50-50." When he was asked whether the greater number of green cabs might indicate a higher probabil for green cabs to be in accidents he responded rather impatiently.

> S: I just told you. There's still 15% chance that the other cabs were causing them, so it doesn't mean that they couldn't cause one. It's 50-50. [19]

Subject #5 was the only subject of the 12 who tried to perform a calculation involving both base-rate and witness information. While his calculations were not appropriate, he seemed to be aware of the importance of the base-rate information for the A and B versions of the problem

> S: ...I would say it would be 70% probability that it was a...uh, green. So it would be 30% probability that it would be a blue cab, I suppose...He's only 80% right. [20]

> I: Mm, Mm.

S: Okay, and...85% of the cabs are blue--er, green... So you have to kind of...you know, balance 80% of 85%... which is...I guess, around 70, you know...it would just seem...So that means it's--you subtract from 100%, that means it's 30%...that are blue. [21]

That answer later seemed low to him, and he brought up his estimate to 60%. When asked why he didn't just say 80% he replied,

S: Because there's more probability that it was a green cab because there are more of them. So you have to count that as a contributing factor...chances are... you get in a hundred accidents, more of them would be green than there would be blue. [22]

Estimates Given the Congruent Version of the Problem

Two subjects (#4 and #6) based their answers to the incongruent versions of the problem exclusively on the base-rate information. The remaining 10 subjects were asked what their estimates would be if the base-rates were reversed, so that not only was the cab identified as blue, but also 85% of the cab accidents involved blue cabs. The Bayesian solution with witness accuracy of 80% is 96%.

Six of the 10 did in fact increase their estimate given the congruent base-rate information. Four of the interviewed subjects gave an answer higher than 85%. It seems clear that some subjects were combining base-rate and witness information.

S#1: Because 85% of the cars are blue...and the lady--80 times said that she was right, more than likely, if I was the judge, I'd say, "You've had it." [23]

I: So that information could affect your estimate of the probability?

S#9: Yeah. It would make me feel, uh...you're thinking of 85%; that's a high percent...and this man saw the same color as that 85%...you've got to believe that uh, you've got to...to think that there's a good probability that it's true. A really good probability. [24]

These two subjects did not say anything to indicate that they had noticed any contradiction between using the base-rate information when it was congruent with the witness information but not when it was incongruent. Subjects #2 and 3 did, however, express their concern. Subject #3 resolved the dilemma by arguing that, in fact, she had taken the base-rate into account initially, and that that was why she had given 80% rather than 100% as an answer. Subject #2 gave a higher answer initially, then said that it didn't make sense.

> S: Because, if I...depend on...well, if I use these if they're switched, why don't I use them when, when they're normal? [25]

He later justified not using the base-rate in either case.

> S: Just because the green cabs get in more accidents doesn't mean that the blue cab couldn't have hit--couldn't have hit the person. [26]

Subject #5, who gave the most Bayesian answer to the incongruent versions did not do so in the congruent case. While he increased his estimate, he put it at 75 or 80%, arguing,

> S: ...he's still only right 80%--he can't be more than 80% because he's only right 80% of the time. [28]

It is also interesting that he saw 80% (and not 85%) as an upper bound, and thus still seemed to maintain a tendency to view the witness information with higher regard than the base rate.

An Approach with a Deterministic Component

While even a clearer case can be made from the interviews than from the questionnaire data that subjects did not approach the cab problem from a Bayesian point of view, it was not immediately apparent how best to describe what they actually did when given the problem. It was clear that not all subjects approached the problem in the same way. Moreover, there were statements that subjects made during the course of the interviews that puzzled us for some time. We finally concluded that one of the approaches to the cab problem was particularly interesting, because not only was it nonBayesian, there seems good reason to characterize it as having a deterministic (nonprobabilistic) component. We judged this approach to be a part of the thinking of most of our subjects, especially subjects #1, #3, #4, and #9.

According to this approach, the primary task of the subject is to form a belief about which cab was actually involved in the accident. This belief is formed through a process in which evidence is evaluated and conflict between sources of evidence is resolved. For incongruent versions of the problem, one source of evidence is considered to be dominant on the basis of such considerations as reliability or relevance, and the other is discredited or assimilated into the belief, so that the strength of belief depends only on the dominant source of information. For example, a subject who decides that the witness report is reliable might argue that the fact there are many more green cabs than blue ones does not preclude the possibility that a blue cab was involved in the accident. We should emphasize that we consider this approach to have a deterministic component because the primary decision is whether a blue cab or green cab was actually involved, not how likely it was that a blue or green cab was involved. The assigning of "probability values" is considered to be secondary, and occurs only after the primary decision has been made. Subjects adopting this approach will sometimes express confusion when asked to give a numerical probability estimate (e.g., see excerpt 31) and when answers are given, they seem to be confidence ratings or indications of strength of belief based on the dominant information.

We were led to consider this type of approach by frequent statements made by subjects about which cab company was actually involved in the accident and whether the witness was correct in his or her identification (e.g., see excerpts 7, 10, 15, 16). This approach could have been elicited by the courtroom context of the problem, but probably is more general, since most real-world tasks involve arriving at a belief or decision rather than assigning probability values (cf. the common understanding of hypothesis testing by social scientists).

The two subjects who did not use the witness information perceived the witness as not being reliable enough. Subjects who considered the witness information to be dominant perceived it as being more relevant than the base-rate inforamtion. Subjects frequently stated that the second source of information was less important or emphasized that this information did not preclude their decision. For example,

> S#7: I'd still say--I'd still keep my answer, even though, uh, more green cabs were involved in accidents. There still were some blue cabs involved in accidents, and it could be that he just happened to observe one of the 15% of the blue cabs. [29]

> S#9: I'd go by the witness again...you know, even though there's a greater amount of accidents...involving green than there are blue...it does not--that does not say that the blue cars--blue cabs will not have accidents...That 15% says that they do have accidents. Maybe not as high as 85% of the green, but they do have accidents... uh...and that could be just one of those...that could be part of the 15% right there. [30]

The logic here thus seems to be that if the evidence does not make it impossible for the cab to be blue, the percentage of blue cabs is irrelevant.

The confidence ratings or indications of strength of belief that subjects provided when asked for probability estimates were sometimes taken directly from the numbers associated with the dominant source of evidence, but in several cases these numbers served only as a guide, S#1: What is the probability? I have to figure it out in percent?

I: Yes. [31]

S: I'd say about 75%...well, more than that...about 80%

or, for example,

I: So what would be your best guess as to the probability that it was a blue cab? You know, in terms of percent.

S#9: I'd give the man an 80.

I: You say 80% chance that it was blue?

S: Mm, mm.

I: Based on...

S: Right there, on that 80.

I: On that 80%. [32]

S: I'd even give him an 85.

I: Why 85?

S: Well, I feel that he did choose the right cab. Not unless he's blind, you know...I'm sure he could tell the difference between blue and green.

also, continuing from excerpt 7,

I: So why didn't you pick just 80?

S#3: Just that he was wrong 20% of the time is a lot... of identifying the difference in two colors. Um, I guess I could have just as well said 80. Some reason I'd just think of, like rounding it off and... three quarters. [33]

Finally, we had wondered how some students could disregard the base-rate information in the incongruent versions, and then use it in the congruent version with apparently no sense that they were being inconsistent. This becomes reasonable, however, from the approach described in this section. When two pieces of evidence are contradictory, one will be discredited or explained away and thus will not influence the confidence rating based on the dominant evidence. However, when both sources agree, they can both be viewed as valid, and thus both can and should be used in the establishment of a confidence rating.

General Discussion

For each of the problems we used, we found that the most common answer seemed to be based on the individuating information. However, the situation is much more complicated than would be implied by any statement to the effect that subjects commit the base-rate fallacy except when the base-rate information can be given a causal interpretation.

We found that responses to the Bayesian probability estimation problems we used were characterized by a great degree of variability. With the exception of the positive test part of the Cancer A problem, the modal answer did not account for the majority of responses. There were strong individual differences in the perceived importance of base-rate information and there seemed to be individual differences in how the probability estimation task itself was interpreted. We found that when the incongruent versions of the cab problem were administered by questionnaire, 47.4% of the responses to the causal version and 45.6% of the responses to the noncausal version were between the base-rate level of 15% and the witness accuracy level of 80%. This seems to suggest that it was not uncommon for subjects to think that both base-rate and witness information were relevant. We do not have any evidence, however, that subjects had any reasonable idea how to combine the two kinds of information.

The interview data suggest that it is often difficult to tell from the answer whether or not the subject thought that the base-rate information was important. It is quite clear that several subjects responding to the cab problem with the modal answer of 80% thought that both the base-rate and witness information were relevant, and reluctantly based their answer on the latter only after they had decided that they had no idea how the two sources of information should be combined. On the other hand, several subjects who based their answers entirely on the witness information gave answers other than 80%. We have indicated earlier how the answers 75%, 85%, and 90% were generated by subjects who ignored the base-rate information.

We found no evidence that the "causality" of base-rate information had a potent effect on the probability estimates given by our subjects. We chose causal and noncausal versions of problems previously used by Ajzen (1977) and Tversky and Kahneman (1980) and found that the mean response and the distribution of responses was virtually the same for the causal and noncausal versions (see Tables 4 and 5). The 12 subjects who were interviewed on the cab problems used by Tversky and Kahneman were given first the noncausal and then the causal version. Most subjects noticed the nature of the change that had been made but only one of the 12 wished to change her answer. It should be noted that the causal base rates we (and Ajzen and Tversky & Kahneman) employed were only "indirectly causal" in that subjects could infer a causal factor that might reasonably by involved but were not told explicitly what it was. For example in the cab problem, subjects were told that one company was involved in 85% of the accidents, allowing the inference that it. drivers were more reckless or incompetent. Subjects were not simply told that the drivers were more reckless. Perhaps base-rate information for which the causal relationship of some interest to note that none of the four subjects we classified as adhering to the deterministic strategy had taken a statistics course.

Finally, there were indications in our data that subjects are more likely to incorporate base-rate information into their answers when it is congruent with individuating information than when it is incongruent. This is seen most clearly in the interview data. Six subjects who were first given incongruent base-rate information increased their estimates when given the congruent base-rate information, and four of them gave an answer larger than 85%. This conclusion is also suggested by questionnaire data. For example, in the Cancer A problem (see Table 3), the base-rate information had a much larger influence on the answer for the negative test result than for the positive test result.

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Footnote

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Table 1

Table of Joint Probabilities for the Cab Problem

		Witness Report b	Witness Report g
Cab	B	.8 x .15 = .12	.2 x .15 = .03
Cab	G	.2 x .85 = .17	.8 x .85 = .68

Table 2

Numbers of Respondents in the Eight Classes

Class	Cancer		Taxicab			Exam	
	A	B	A	B	C	A	B
1	28		15	13			
2	36		20	16			
3	43		17	16		16	17
4			11	11		11	11
5			18	18		18	18
6		20			20		
7		7			8		
8		16			19		

Table 3

Questionnaire Responses to the Cancer Problem

P (cancer/positive test result)

Problem	Correct Answer	Mean Response	Median Response	Modal Response	Distribution of Responses 95%	>95%	<95%	Total Number of Responses
Cancer A	16%	83.4%	95%	95%	56 (75.6%)	7 (9.5%)	11 (14.9%)	74
Cancer B	1.9%	47.5%	10%	95%	16 (43.2%)	1 (2.7%)	20 (54.1%)	37

P (cancer/negative test result)

Problem	Correct Answer	Mean Response	Median Response	Modal Response	>5%	5%	Distribution of Responses 1-5%	1%	.1-1%	.1%	<.1%	Total Number of Responses
Cancer A	.05%	14.1%	5%	5%	10 (13.5%)	31 (41.9%)	0 (0)	10 (13.5%)	9 (12.2%)	2 (2.7%)	12 (16.2%)	74
Cancer B	.005%	8.3%	5%	5%	5 (13.5%)	15 (40.5%)	1 (2.7%)	1 (2.7%)	3 (8.1%)	1 (2.7%)	10 (27.0%)	37

Table 4

Responses to the Taxicab A (noncausal) and B (causal) Problems
(Bayesian Answer = 41%)

Class	Form	Mean	<15%	15%[a]	between 15% and 80%	80%[b]	>80%	No Answer	Total
1	causal	63.5%	0	0	10	2	3	1	15
	noncausal	68.9%	0	0	5	8	0	0	13
2	causal	74.5%	0	0	4	8	2	1	16
	noncausal	72.0%	0	0	7	8	5	0	20
3	causal	68.4%	1	0	8	4	2	0	16
	noncausal	67.3%	0	0	9	5	3	0	17
4	causal	46.9%	0	3	6	2	0	0	11
	noncausal	66.6%	0	1	4	5	1	0	11
5	causal	68.1%	0	0	8	8	1	2	18
	noncausal	67.4%	0	0	11	7	0	0	18
Total	causal	65.8%	2 (2.6%)	3 (3.9%)	36 (47.4%)	24 (31.6%)	7 (9.2%)	4 (5.3%)	76
	noncausal	68.7%	0 (0%)	1 (1.3%)	36 (45.6%)	33 (41.8%)	9 (11.4%)	0 (0%)	79

[a] base rate

[b] witness accuracy

Table 5.

Responses to the Examination Problems

Class	Form	Mean	<50%	between 50% and 75%	75%[a]	>75%	No answer	Total
3	causal	55.0%	5	8	2	1	1	17
	noncausal	56.0%	4	9	0	2	1	16
4	causal	65.0%	1	4	6	0	0	11
	noncausal	63.6%	2	2	7	0	0	11
5	causal	48.7%	9	7	0	2	0	18
	noncausal	55.6%	6	9	1	2	0	18
Total	causal	54.9%	15 (32.6%)	19 (41.3%)	8 (17.4%)	3 (6.5%)	1 (2.2%)	46
	noncausal	57.7%	12 (26.7%)	20 (44.4%)	8 (17.8%)	4 (8.9%)	1 (2.2%)	45

[a]base rate

Table 6

Responses to the Taxicab C Problem
(Bayesian Answer = 93%)

Class	Mean	<50%	between 50% and 70%	70%[a]	between 70% and 85%	85%[b]	>85%	No Answer	Tota
6	69.6%	0	5	8	3	0	2	2	20
7	65.0%	0	4	1	2	0	1	0	8
8	67.1%	0	9	3	4	1	0	2	19
Total	67.8%	0	18 (38.3%)	12 (25.5%)	9 (19.1%)	1 (2.1%)	3 (6.4%)	4 (8.5%)	47

[a]witness accuracy

[b]base rate

Table 7

Answers of Subjects Interviewed on Taxicab Problems

Subject Number	Sex	Version of Problem Noncausal	Causal	Congruent	Comments	College Math or Statistics
1	F	80	80	95	a	calculus
2	M	90	90	90/95		calculus
3	F	75/80	80	90	a	none
4	F	15	15			precalculus*
5	M	30/60	60	75/80		calculus
6	F	15	15			intro statistics
7	F	80	80	80	a	intro statistics/ calculus
8	M	50	50	50		calculus*
9	M	80/85	80/85	90		calculus
10	F	80	80	80		none
11	F	80	50	80	b	intro statistics
12	F	80	80	80	a,b	intro statistics

* currently enrolled

a. spontaneously commented that if there was no witness information they would use base-rate information

b. commented that there was a way to combine the information, but that they did not know how to do it