TEACHING CONCEPTS RATHER THAN CONVENTIONS

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The computer's potential to improve the teaching of data analysis is now a well-known litany (Jones, 1997; Snell & Peterson, 1992; Velleman & Moore, 1998). It includes its power to illuminate key concepts through simulations and multiple-linked representations. It also includes its ability to free students up, at the appropriate time, from time-intensive tasks—from what NCTM's (1989) Standards referred to as the "narrow aspects of statistics" (p. 113). This potentially allows instruction to focus more attention on the processes of data analysis—exploring substantive questions of interest, searching for and interpreting patterns and trends in data, and communicating findings.

However, as Biehler (1995, p.3) has suggested, the younger the student, the more difficult it is to design an appropriate tool for learning statistics. Most of the existing tools for young students have been developed from the "top down." They provide a subset of conventional plots and thus are simpler than professional tools only in that they have fewer options. These "simplified professional tools" are ill-suited to younger students who "need a tool that is designed from their bottom-up perspective of statistical novices and can develop in various ways into a full professional tool (not vice versa)."

Tinkerplots is a data analysis tool for the middle school that we are designing "from the bottom up" (Konold & Miller, 2001). When a data set is first opened in Tinkerplots, a plot window appears showing a haphazard arrangement of data icons on the screen. As in Tabletop (see Hancock, Kaput & Goldsmith, 1992), each icon represents an individual case. But in Tinkerplots, rather than choosing from a menu of existing plot types (e.g., bar graph, pie chart, scatterplot), students progressively organize the data using a small set of intuitive operators including "stack," "order," and "separate." By using these basic operators in different combinations, students can construct a large variety of graphical representations. These include many of the standard graphs but also many we have never seen before. Thus, as the name suggests, Tinkerplots is a plot construction set that students can use to design their own graphs.

Using Tinkerplots, students can begin exploring data without knowing the difference between various data types (nominal, ordinal, ratio), without an explicit understanding of the difference between characteristics (tall) vs. variables (height), and without knowledge of the conventions of 2-D representations. Our hope is that by using this tool, students can systematically build up their understandings of various displays and the statistical ideas they embody.

To demonstrate features of Tinkerplots that allow students to use what they already know to analyze data, I consider various ways of displaying the relation between two variables. Using these alternative representations, even young students appear to be able to make sound judgments about covariation. I use this example to argue that we should interpret Biehler's (1995) critique contrasting "bottom up" vs. "top down" design more generally, because current instructional objectives in data analysis, like data analysis software, tend to be constructed in top-down fashion based on expert practice. In general, our approach has been to take the set of displays and other statistical tools that statisticians have traditionally used, and divvy them up among the grades according to our sense of their difficulty. So grades 3-5 get line graphs and medians, grades 6-8 get scatterplots and means, and grades 9-12 get regression lines and sampling distributions (see NCTM, 2000). A bottom-up approach would, instead, structure objectives according to how statistical reasoning develops in young students. To be fair, until now we have had little research to inform us on how young students reason about data, but this is quickly changing (see, for example, Lajoie, 1998; Konold & Higgins, in press; Lehrer & Schauble, 2002.)

Judging Covariation from Standard Displays

Research suggests that even adults have difficulty making judgments about covariation from standard representations, from contingency tables in the case of two qualitative variables and from scatterplots in the case of two numeric variables (see Batanero, Estepa, & Godino, 1997; Konold & Higgins, in press). We might be tempted to interpret this research as suggesting that people are poor at judging covariation. However, we would hardly be able to function in our environments if we did not notice and make reasonable judgments about relations among phenomena: The louder something sounds, the closer it tends to be; the more one practices, the better one gets. This suggests the possibility that people do not have difficulty judging covariation per se but rather they have difficulty decoding displays such as scatterplots and contingency tables.

Some evidence for this possibility comes from recent research by Noss, Pozzi, and Hoyles (1999) who studied the statistical reasoning of practicing nurses. After receiving instruction on using scatterplots, the nurses analyzed a health data base of British adults to explore the relationship between age and blood pressure. The nurses knew from experience that with increasing age, blood pressure tends to rise, which is what the data they were analyzing showed as well. Using statistical software, the nurses quickly generated a scatterplot of age and blood pressure, similar to the one shown in Figure 1. However, they were unable to see evidence of the positive relationship between age and blood pressure that was there in the scatterplot.

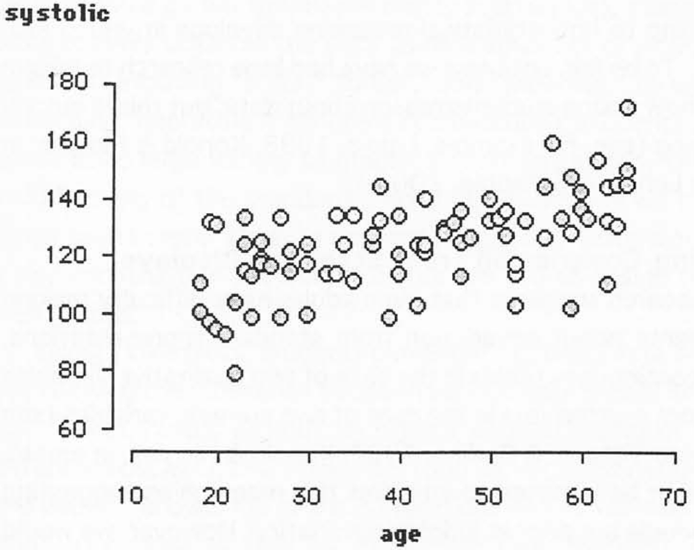

Figure 1
Scatterplot of age and systolic blood pressure, adapted from data in Noss, Pozzi, and Hoyles (1999)

Below I demonstrate with Tinkerplots three alternative ways of representing the relation between two numeric variables that appear to be easier for students to interpret than are scatterplots. More generally, I hope to demonstrate the value of students having access to a data analysis tool that allows them more flexibility in designing data displays than do most existing tools.

Scatterplot Slices

In their study, Noss, Pozzi, and Hoyles (1999) prompted the nurses to make a display similar to the one shown in Figure 2. This display reorganized the continuous age variable into several categories (ages 15-29, 30-44, 45-59, 60-75).

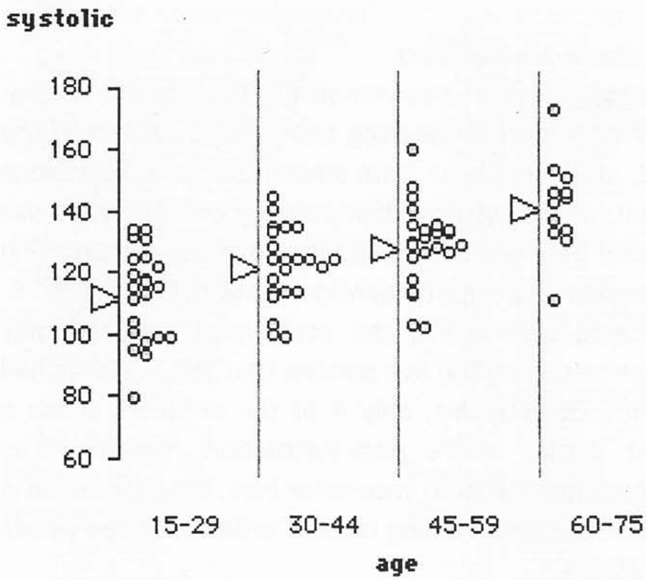

Figure 2

Systolic blood pressure of people grouped in various age categories. Adapted from Noss, Pozzi, and Hoyles (1999). The triangles show the location of the mean blood pressure in each age grouping.

Cobb, McClain, and Gravemeijer (in press) have used this same type of representation in their teaching experiments at the middle school. Their hope is that by seeing each vertical slice of data in this representation as a distribution of a discrete group, students can apply skills they have learned in comparing two distributions to visually compare the centers of the distributions in the "sliced" scatterplot. This is precisely what Noss et al. (1999) observed the nurses in their study doing. Using the sliced scatterplot, the nurses computed the average blood pressure in each of the age groupings. Based on an analysis of these averages, the nurses were able to see the expected trend of higher blood pressures associated with increased age.

Ordered Case-Value Bars

Cobb et al. (in press) interviewed 11 students who participated in their 8th grade teaching experiment. As part of the interviews, students chose from among five representations the one that would best allow them to judge whether there was a relationship between two variables (e.g., between brushing time and amount of plaque remaining on teeth). During the 14-week teaching experiment, the scatterplot was the only representation among the five options that the students had worked with. Despite this, only 4 of the students chose a scatterplot display in the post-instruction interview. The others chose either ordered case-value bars, like that shown in Figure 3, or the corresponding table of ordered, paired values (Cortina, 2000).

Here is how one of the students explained why she preferred either the paired case-value plot or the ordered table of values to evaluate the relation between brushing time and plaque:

> M: You can follow the numbers. You know that they're getting more time so, so you just look at the percentage [of plague] and see if it increases or decreases.
>
> I: What does it tell you that this one [the scatterplot] doesn't?
>
> M: It's the time in order, instead of just, I don't know, everywhere.
>
> I: Why is that helpful to you, to have the time in order?
>
> M: ... Cause you know it's going to be in order. ... You don't have to keep looking at the [brushing] time.

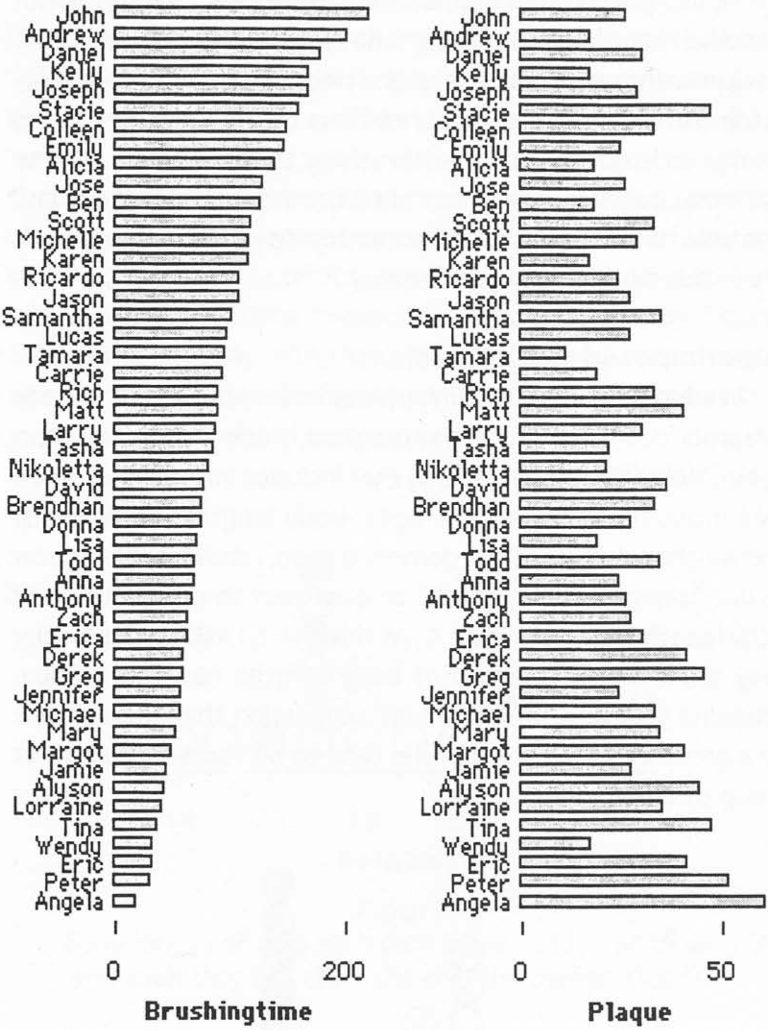

Figure 3

Paired case-value plots showing time in seconds spent brushing (left) and percent of remaining plaque (right).

Knowing that the bars are ordered according to brushing time, you can determine the effects of increasing brushing time by tracking the changing lengths of the "plaque" bars as you visually scan them from bottom to top.

M, along with other students who were interviewed, did not perceive the values of brushing time as being ordered when they were presented in the scatterplot. The paired case-value display made the ordering of the data obvious to her. With the cases clearly ordered according to brushing time, M knew how to systematically scan the values of plaque in search of a trend and was able to offer an accurate summary: "overall the longer you brush the more plaque you destroy."

Superimposed Color Gradient

We have field tested Tinkerplots in several 5th - 7th grade classrooms. To introduce the program, I use a data set from Rubin, Mokros, and Friel (1996) that includes information on 24 cats including their genders, ages, body lengths, tail lengths, and weights. As part of this demonstration, I show students how to use "separate" and "stack" to construct the distribution of body length shown in Figure 4. At this point, I ask students why they think the distribution of body lengths has two humps. Students typically offer as a first suggestion that there might be a group of younger cats, who tend to be short, and another group of older, longer, cats.

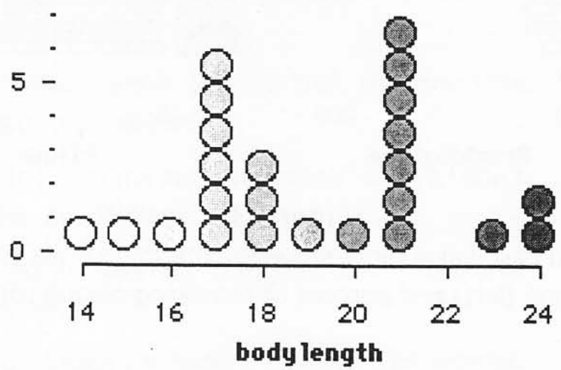

Figure 4
Body length of 24 cats with data icons shaded accorded to body length such that the longer the cat, the darker its icon.

In Figure 4, the case icons are colored according to body length such that the longer the cat, the deeper grey the icon. (What appear in these figures as grey scales appear in Tinkerplots as color gradients, with a different color for each variable.) To test the possibility that age is related to body length, I suggest that we change the color of the icons to show age, and ask the students what they expect to observe. They correctly anticipate that if they are correct, the darker icons associated with the older cats will cluster towards the right of the display. Figure 5 shows the display with the data icons colored according to age. Looking at this display, students are quick to reject the idea that age is related to body length.

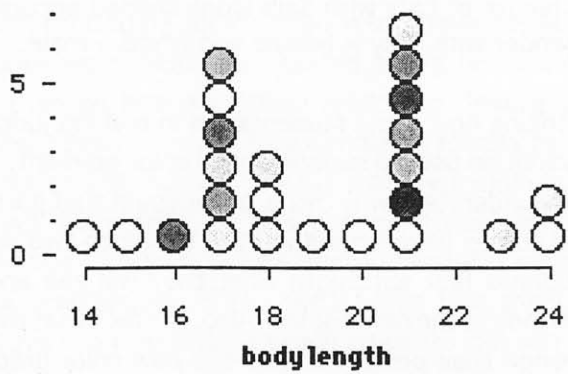

Figure 5
Body length of cats with data icons shaded according to age such that the older the cat, the darker its icon.

At this point, another student will suggest that perhaps female cats are shorter than male cats and will ask that we change the color gradient to show gender. Figure 6 shows the resultant display.

Looking at this plot, students conclude that indeed the male cats tend to be longer than the female cats and that this explains in part why the distribution of body length has two humps.

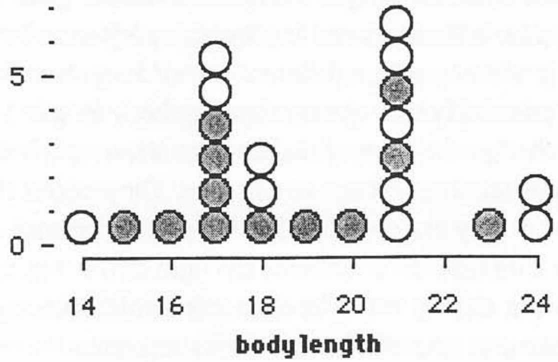

Figure 6
Body length of cats with data icons shaded according to gender with grey = female and white = male.

It is striking how facile students are in making judgments about covariation using a superimposed color gradient. We do not yet fully understand why this is, but suspect that part of the advantage comes from separating the task into two discrete steps: students first anticipate what they will see and then afterwards look at the new display. Also, the fact that the icons do not change their positions when the new color gradient is added probably helps students keep in mind that the cases are still ordered according to the original variable, which then allows them to systematically scan the cases in search of a pattern.

Conclusion

In explaining why even after instruction students have considerable difficulty interpreting what seem like simple plots (such as frequency bar graphs and histograms), Bright and Friel (1998) pointed out how even these plots represent several levels of abstraction. These levels are described by Roth and Bowen (1994) who pointed out that as we move from representing data with maps, to lists, graphs, and finally equations, we move along a continuum of concrete to increasingly abstract statistical representations. As the conventions we use to represent data become more abstract, our displays become more convincing and useful to those who understand them, but harder for novices to interpret. Because with Tinkerplots graphs are built up (or deconstructed) in stages, students are less likely to get disoriented by "imposed" abstraction. They can work from the bottom up, building on the foundation of what they already understand.

However, we should interpret Biehler's (1995) critique contrasting "bottom up" vs. "top down" design more generally, because it is also true that current instructional objectives, like software, tend to be constructed in top-down fashion. Current objectives are typically formulated based on expert practice rather than on how statistical reasoning develops in young students. It seems unwise, for example, to specify as do NCTM'S Principles and Standards (2000) that by middle school, students will learn how to "make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots..." (p. 248). Given that there are many alternative representations which students can use to explore and express ideas such as covariation, spread, center, and shape, we would do better to target the underlying concepts in our instructional objectives and to focus less on how students should represent these graphically.

Acknowledgements

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